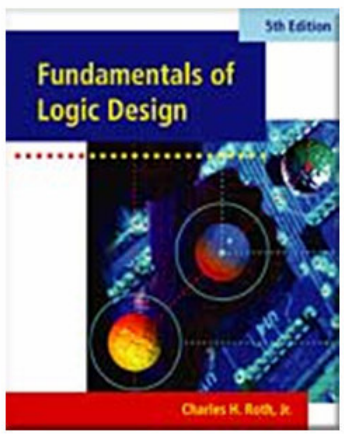
CHAPTER 2

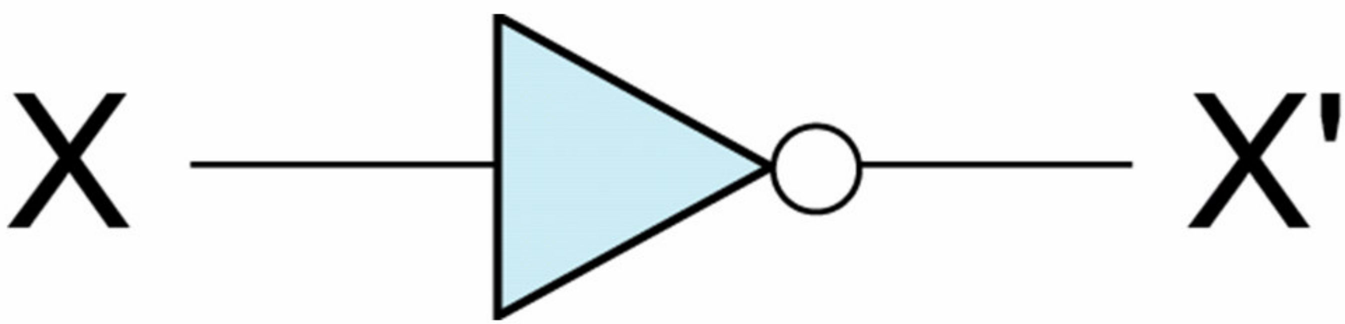
BOOLEAN ALGEBRA



This chapter in the book includes:

$X' = 1$ if $X = 0$

$X' = 0$ if $X = 1$



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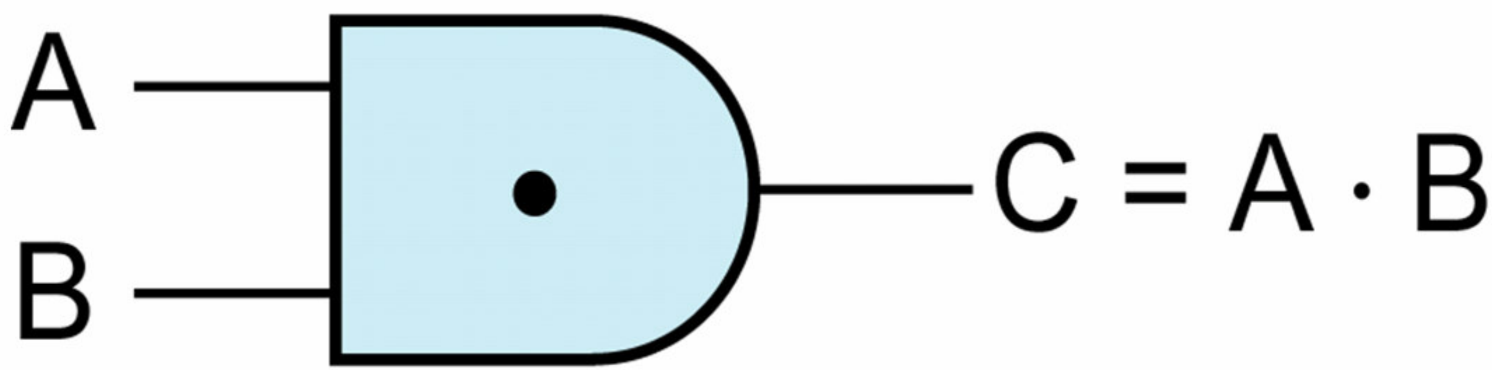
A B	$C = A \cdot B$
0 0	0
0 1	0
1 0	0
1 1	1



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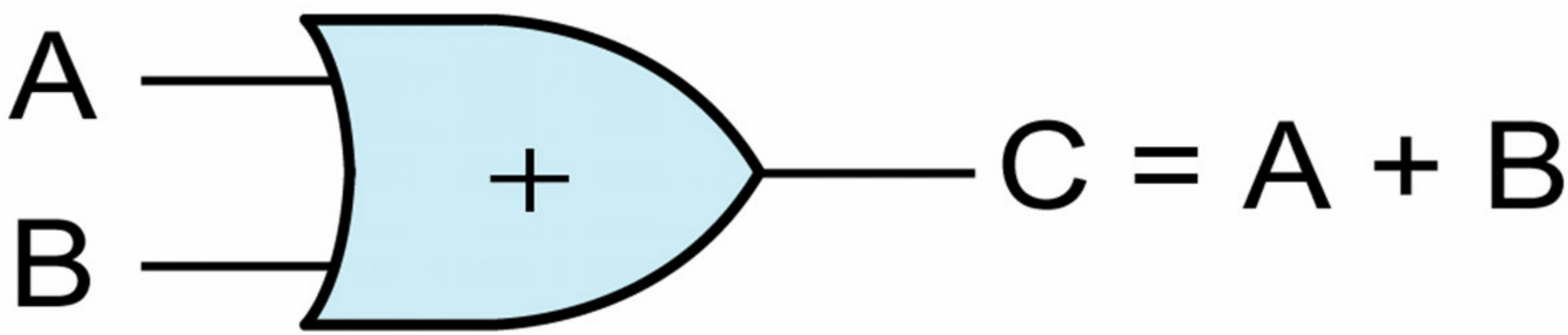
A B	C = A + B
0 0	0
0 1	1
1 0	1
1 1	1



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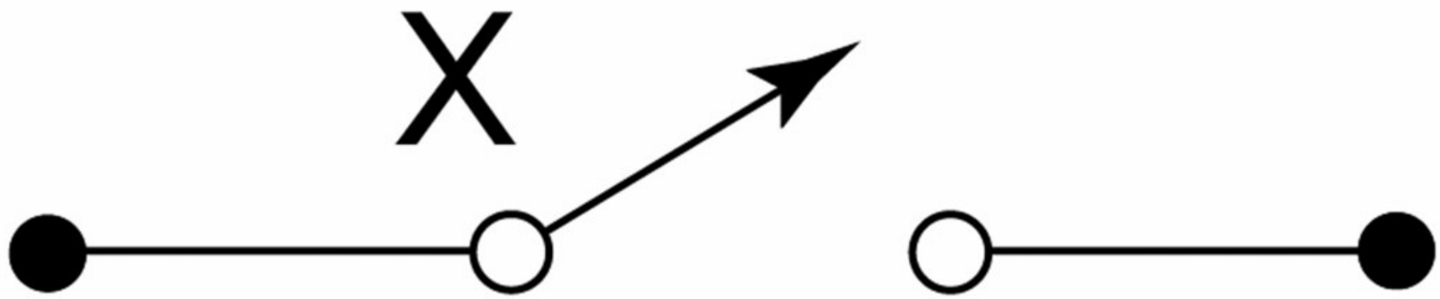
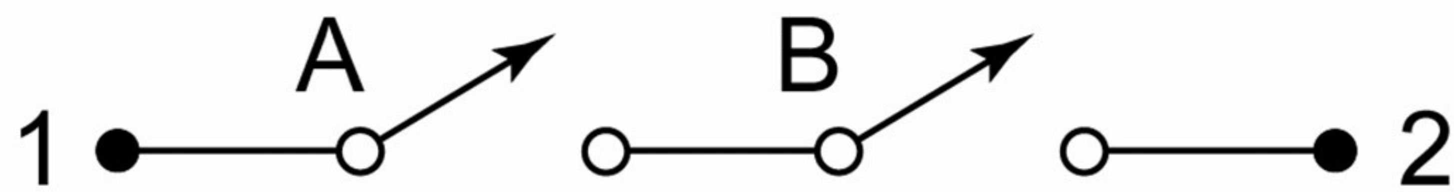


$X = 0$ switch open

$X = 1$ switch closed

Section 2.2, p. 35



T = 0   open circuit between terminals 1 and 2

T = 1   closed circuit between terminals 1 and 2

$$T = AB$$

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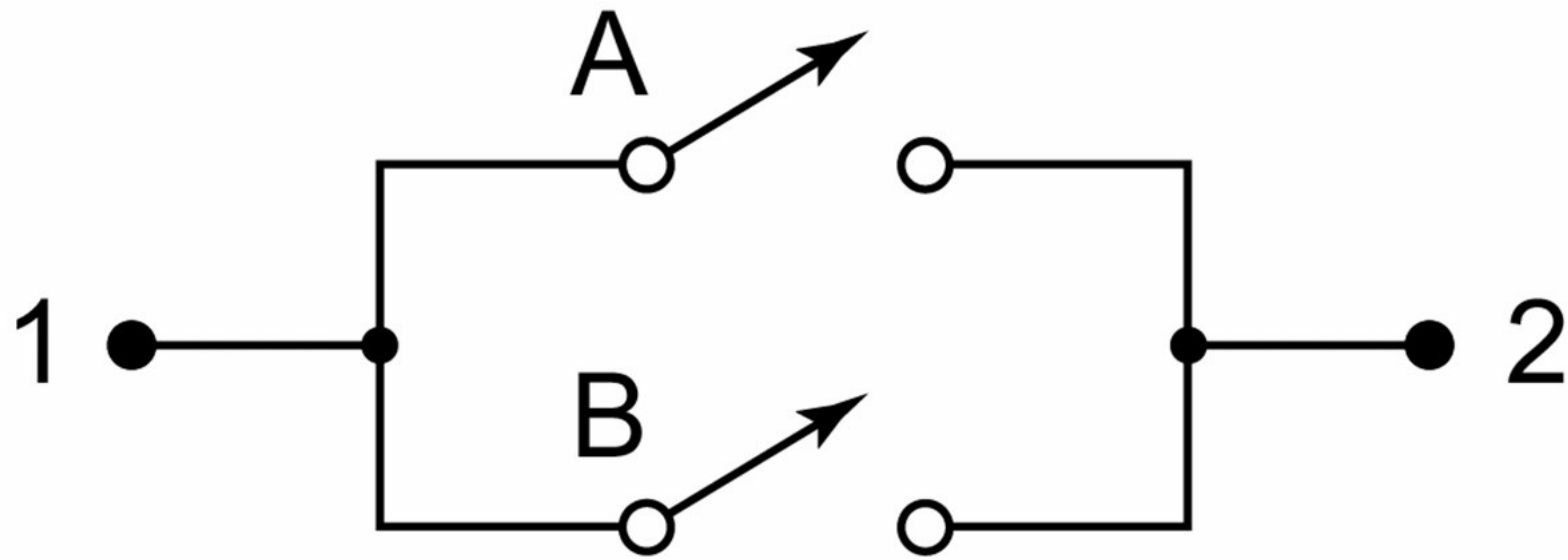

T = A+B

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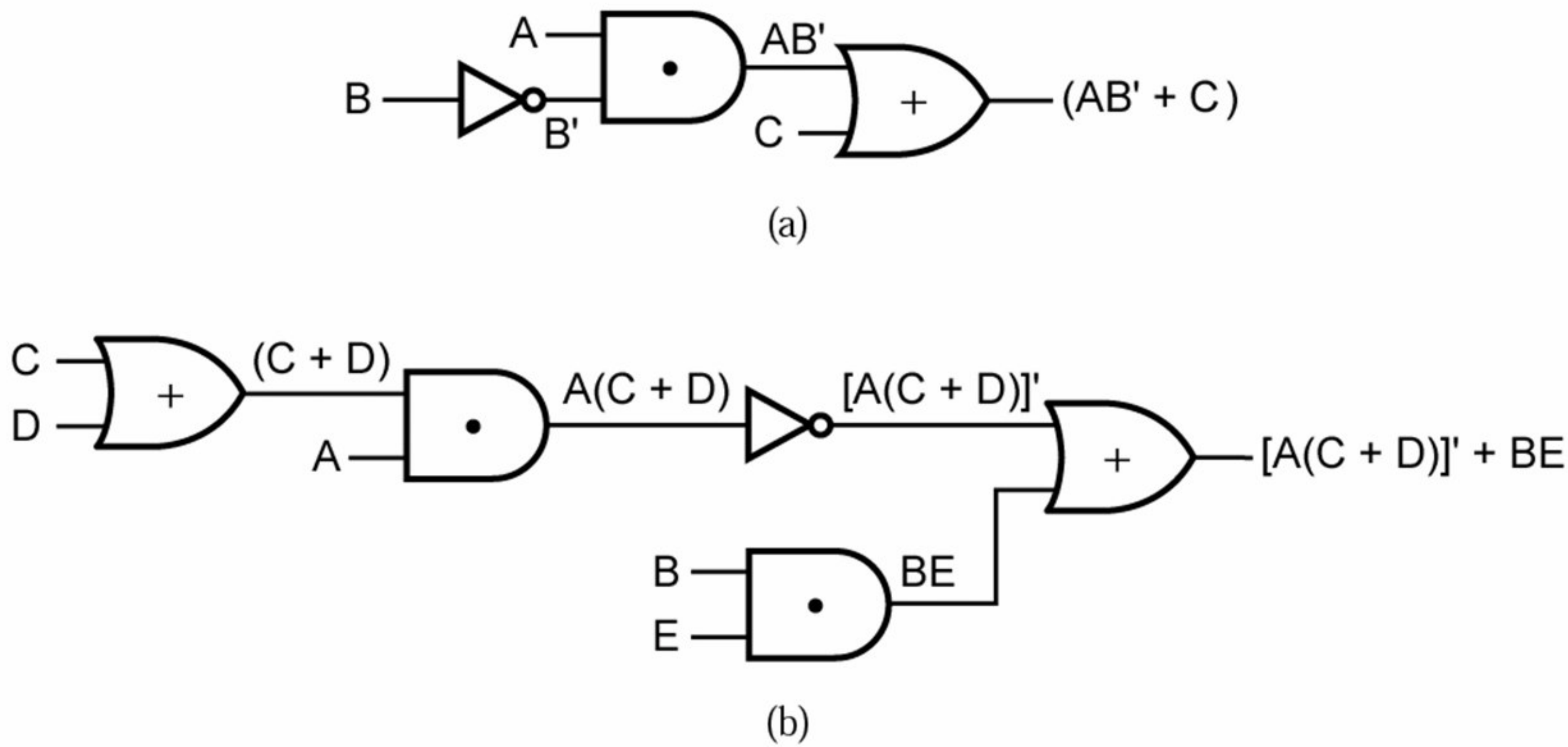
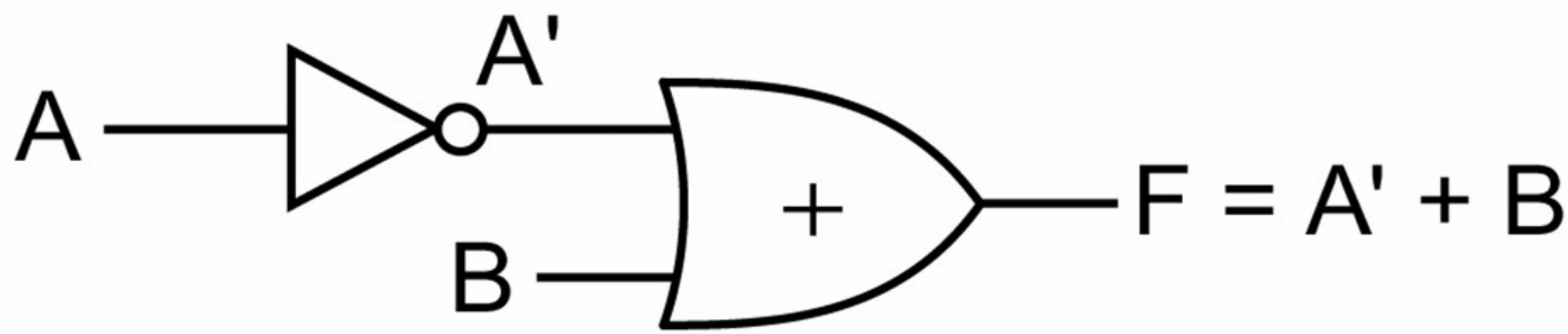


Figure 2-1: Circuits for Expressions (2-1) and (2-2)



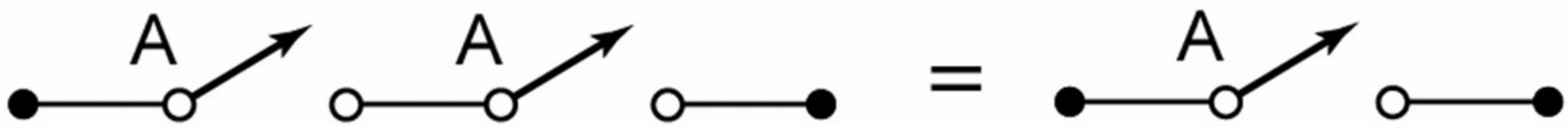
(b)

A B	A'	F = A' + B
0 0	1	1
0 1	1	1
1 0	0	0
1 1	0	1

Figure 2-2: **2-Input Circuit**

Table 2-1: Truth Table for 3 variables

A B C	B’	AB’	AB’+C	A+C	B’+C	(A+C)(B’+C)
0 0 0	1	0	0	0	1	0
0 0 1	1	0	1	1	1	1
0 1 0	0	0	0	0	0	0
0 1 1	0	0	1	1	1	1
1 0 0	1	1	1	1	1	1
1 0 1	1	1	1	1	1	1
1 1 0	0	0	0	1	0	0
1 1 1	0	0	1	1	1	1



$$(A \cdot A = A)$$

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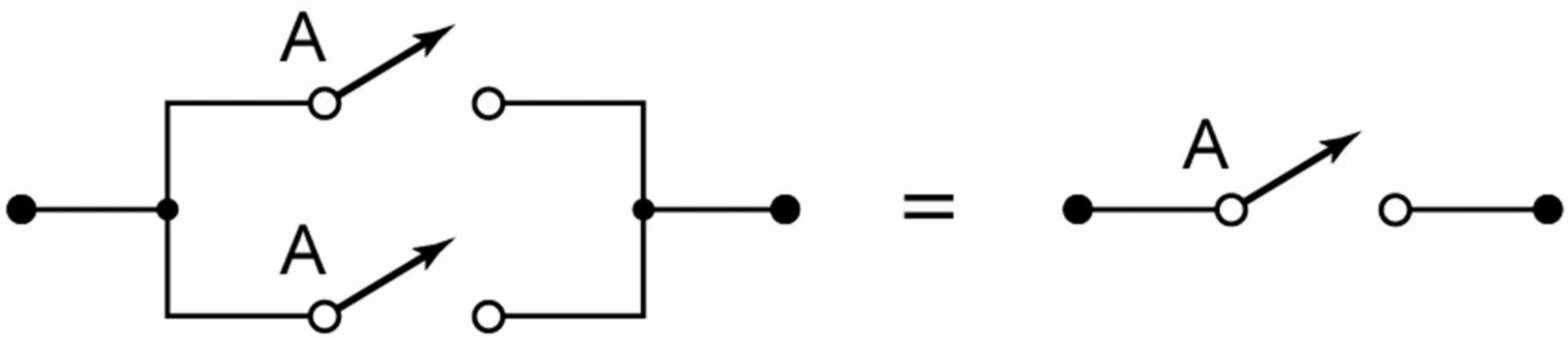
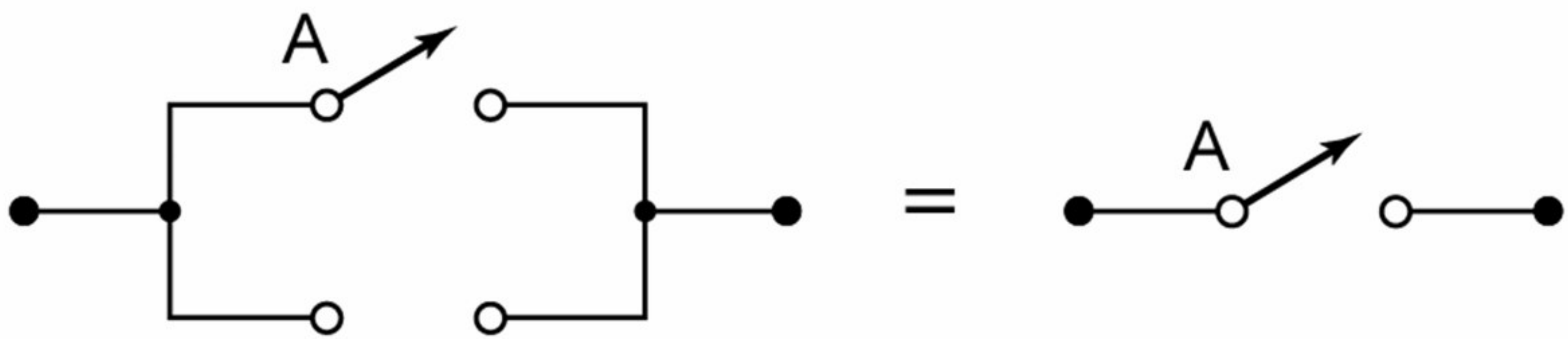


$(A + A = A)$

Section 2.4, p. 38



(A + 0 = A)

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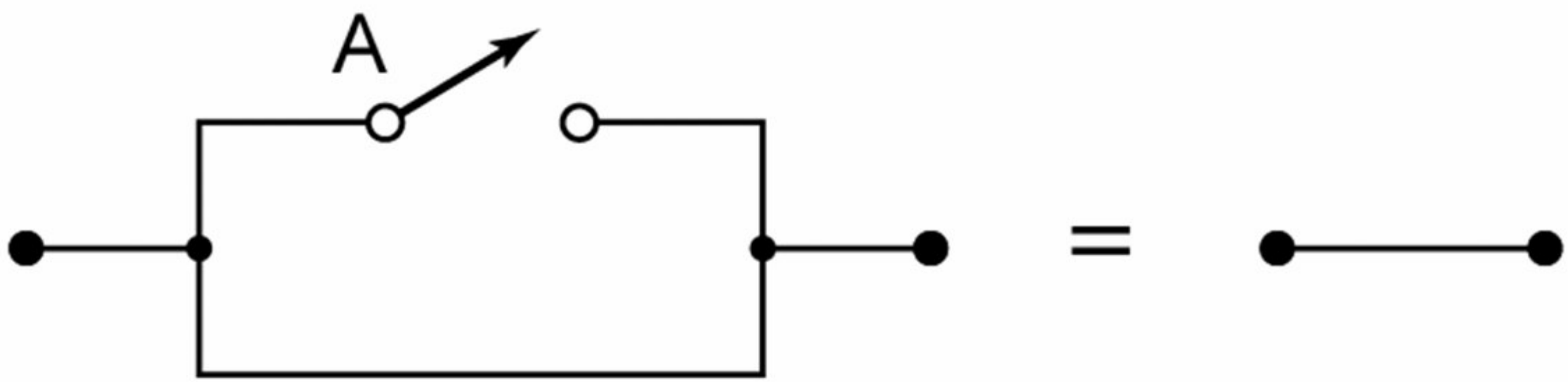

$(A + 1 = 1)$

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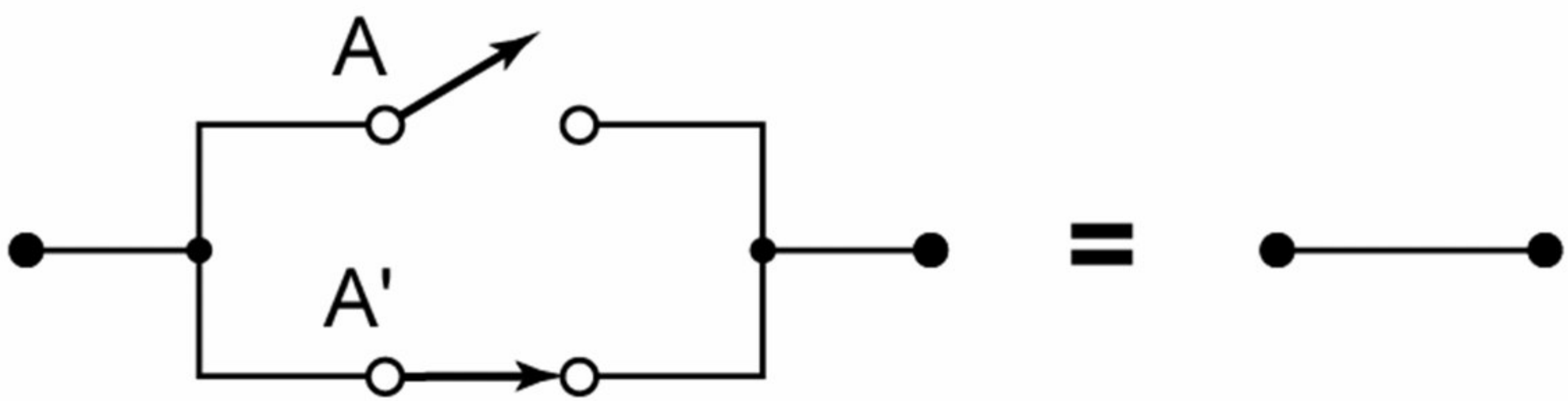

$(A + A' = 1)$

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$(A \cdot A' = 0)$

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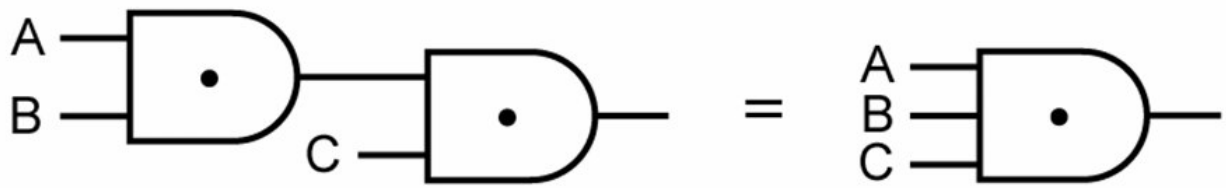
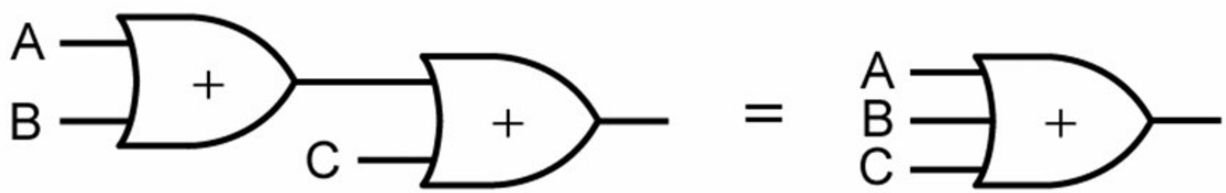
Table 2-2: Proof of Associative Law for AND

X	Y	Z	XY	YZ	$(XY)Z$	$X(YZ)$
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	0	0
0	1	1	0	1	0	0
1	0	0	0	0	0	0
1	0	1	0	0	0	0
1	1	0	1	0	0	0
1	1	1	1	1	1	1



(AB) C = ABC

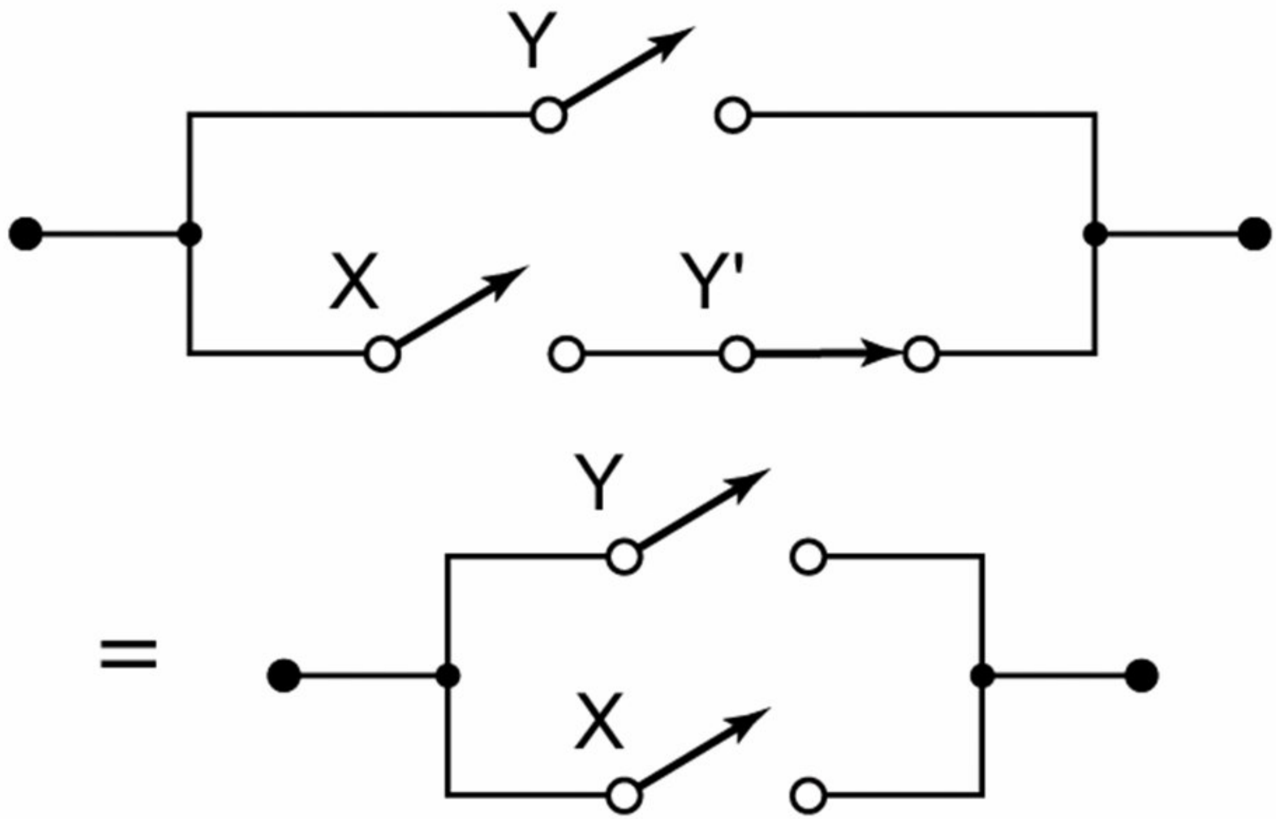
(a)



(A + B) + C = A + B + C

(b)

Figure 2-3: **Associative Law for AND and OR**



$$(Y+XY' = Y+X)$$

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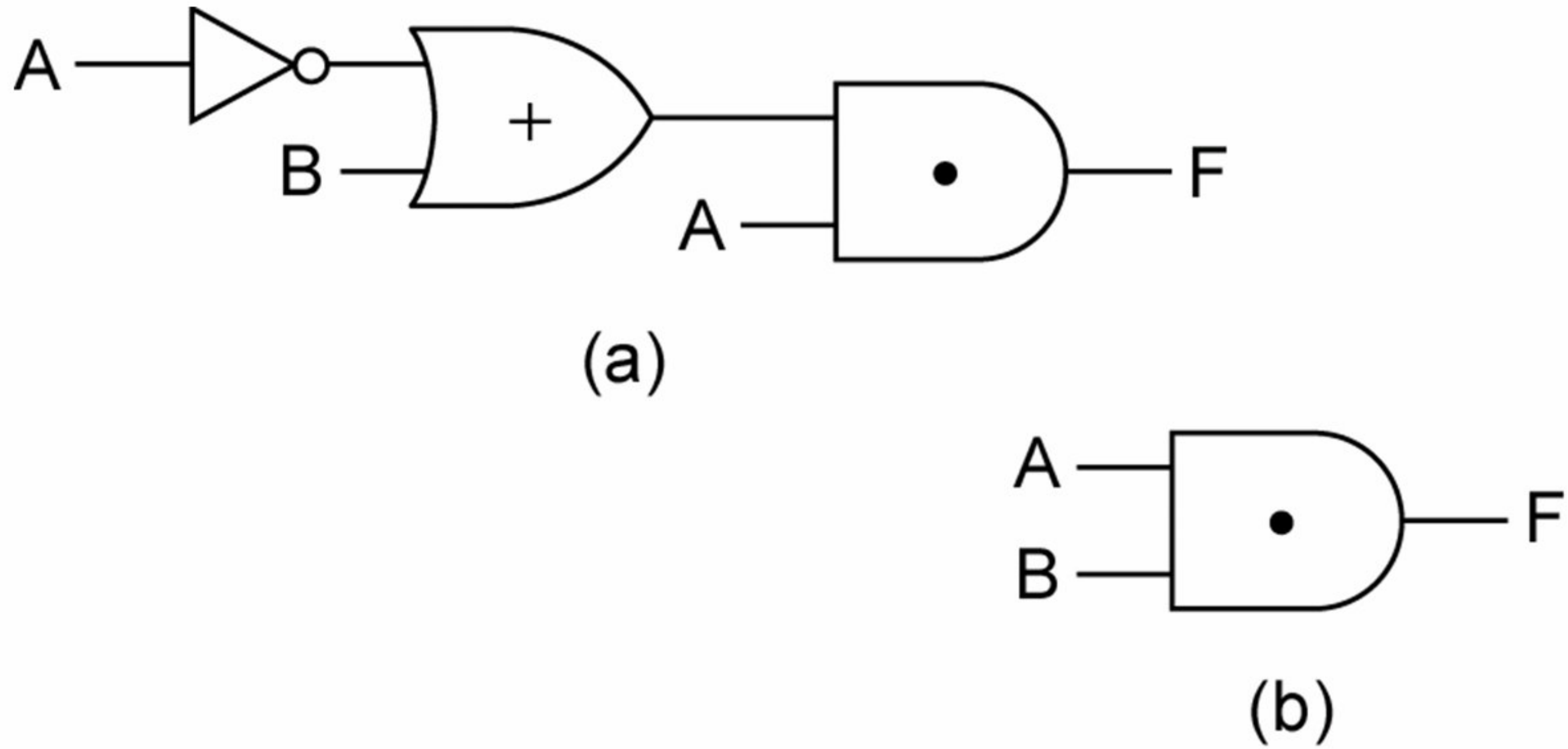

Figure 2-4: **Equivalent Gate Circuits**

EXAMPLE 1 Simplify $Z = A'BC + A'$

This expression has the same form as (2-13) if we let $X = A'$ and $Y = BC$

Therefore, the expression simplifies to $Z = X + XY = X = A'$.

EXAMPLE 2 Simplify $Z = [\underbrace{A + B'C}_{X} + \underbrace{D + EF}_{Y}][\underbrace{A + B'C}_{X} + (\underbrace{D + EF}_{Y})']$

Substituting: $Z = [\ X + Y\ ][\ X + Y'\ ]$

Then, by (2–12D), the expression reduces to

$$Z = X = A + B'C$$

EXAMPLE 3 Simplify $Z = (\underbrace{AB + C}_{Y'})(\underbrace{B'D + C'E'}_{X}) + (\underbrace{AB + C}_{Y})'$

Substituting: $Z = \quad Y' \quad X \quad + \quad Y$

By (2–14D): $Z = X + Y = B'D + C'E' + (AB + C)'$

Simplify (p. 42-43)

EXAMPLE 1: Factor $A + B'CD$. This is of the form $X + YZ$ where $X = A$, $Y = B'$, and $Z = CD$, so

$$A + B'CD = (X + Y)(X + Z) = (A + B')(A + CD)$$

$A + CD$ can be factored again using the second distributive law, so

$$A + B'CD = (A + B')(A + C)(A + D)$$

EXAMPLE 2: Factor $AB' + C'D$

$AB' + C'D = (AB' + C')(AB' + D)$ ← note how $X + YZ = (X + Y)(X + Z)$ was applied here

$= (A + C')(B' + C')(A + D)(B' + D)$ ← the second distributive law was applied again to each term

EXAMPLE 3: Factor $C'D + C'E' + G'H$

$C'D + C'E' + G'H = C'(D + E') + G'H$ ← first apply the ordinary distributive law, $XY + XZ = X(Y + Z)$

$= (C' + G'H)(D + E' + G'H)$ ← then apply the second distributive law

$= (C' + G')(C' + H)(D + E' + G')(D + E' + H)$ ← now identify X, Y, and Z in each expression and complete the factoring

Factor (p. 44-45)

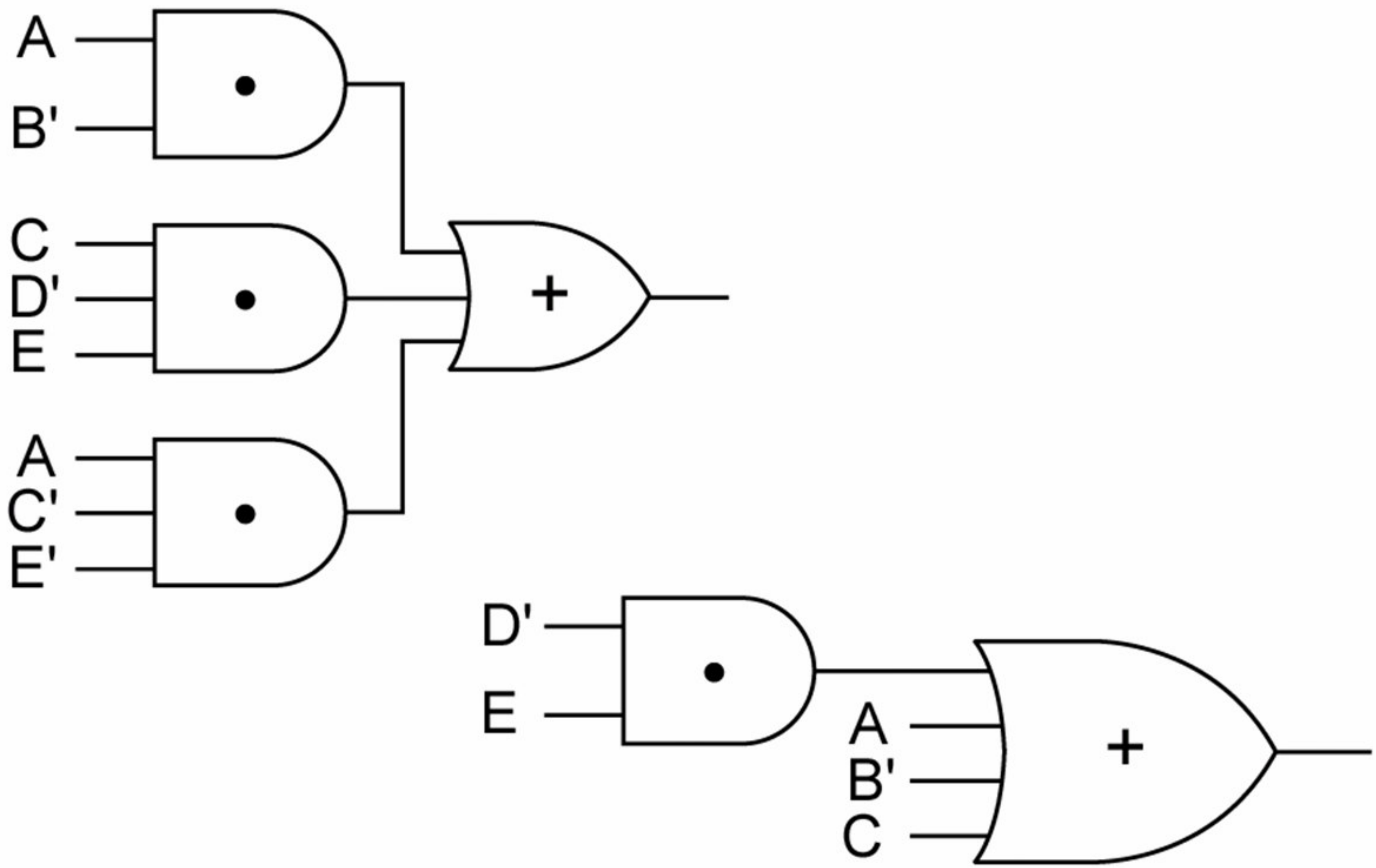

Figure 2-5: **Circuits for Equations (2-15) and (2-17)**

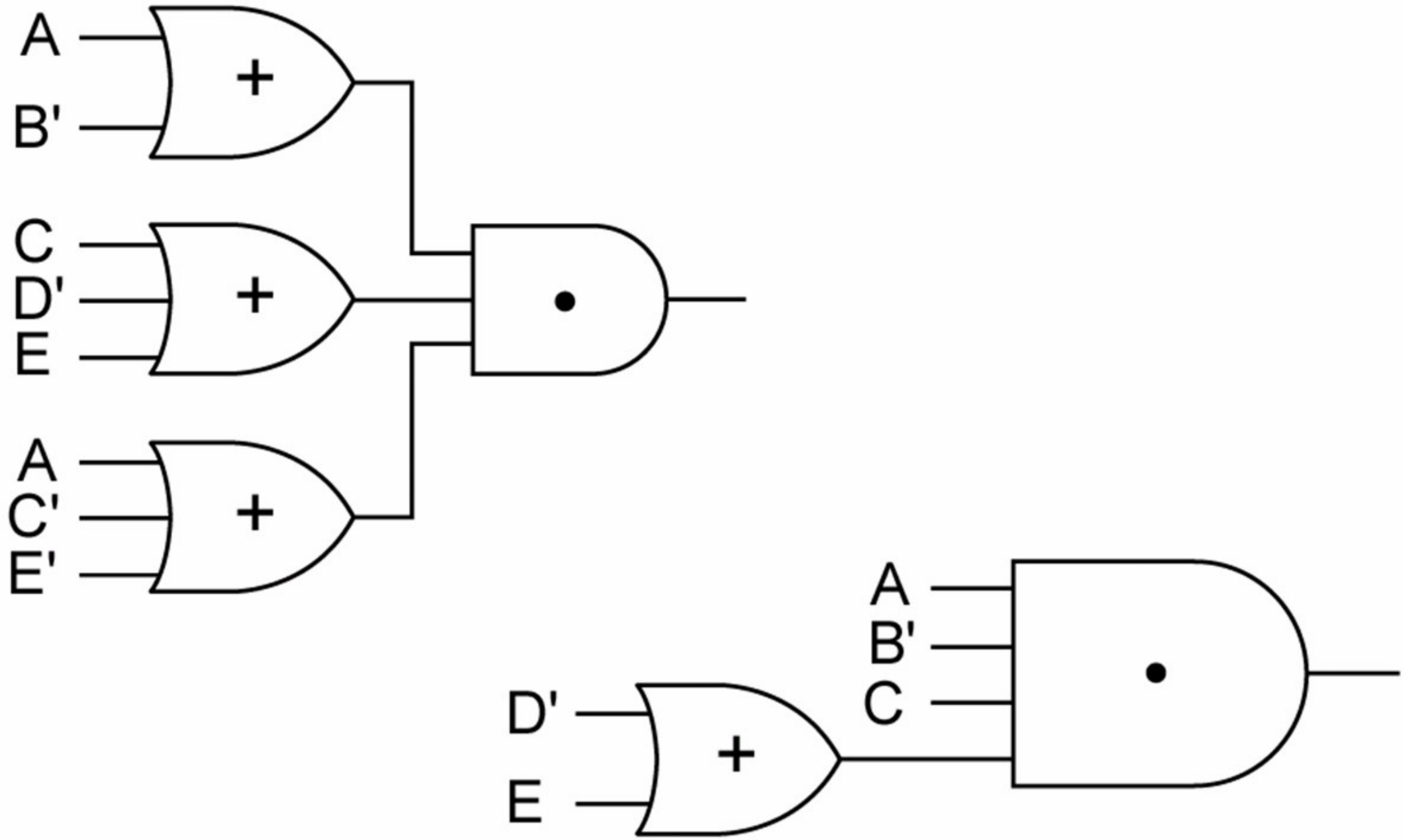

Figure 2-6: **Circuits for Equations (2-18) and (2-20)**

LAWS AND THEOREMS (a)

Operations with 0 and 1:

1. $X + 0 = X$ 1D. $X \bullet 1 = X$
2. $X + 1 = 1$ 2D. $X \bullet 0 = 0$

Idempotent laws:

3. $X + X = X$ 3D. $X \bullet X = X$

Involution law:

4. $(X')' = X$

Laws of complementarity:

5. $X + X' = 1$ 5D. $X \bullet X' = 0$

LAWS AND THEOREMS (b)

Commutative laws:

6. $X + Y = Y + X$ 6D. $XY = YX$

Associative laws:

7. $(X + Y) + Z = X + (Y + Z) = X + Y + Z$ 7D. $(XY)Z = X(YZ) = XYZ$

Distributive laws:

8. $X(Y + Z) = XY + XZ$ 8D. $X + YZ = (X + Y)(X + Z)$

Simplification theorems:

9. $XY + XY' = X$ 9D. $(X + Y)(X + Y') = X$
10. $X + XY = X$ 10D. $X(X + Y) = X$
11. $(X + Y')Y = XY$ 11D. $XY' + Y = X + Y$

LAWS AND THEOREMS (c)

DeMorgan's laws:

12. $(X + Y + Z + \ldots)' = X'Y'Z'\ldots$ 12D. $(XYZ\ldots)' = X' + Y' + Z' + \ldots$

Duality:

13. $(X + Y + Z + \ldots)^D = XYZ\ldots$ 13D. $(XYZ\ldots)^D = X + Y + Z + \ldots$

Theorem for multiplying out and factoring:

14. $(X + Y)(X' + Z) = XZ + X'Y$ 14D. $XY + X'Z = (X + Z)(X' + Y)$

Consensus theorem:

15. $XY + YZ + X'Z = XY + X'Z$ 15D. $(X + Y)(Y + Z)(X' + Z) = (X + Y)(X' + Z)$